(1) Prove the following by the Principle of Mathematical Induction.

I. $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$

II. $1\cdot2+2\cdot2^2+3\cdot2^3+\cdots+n\cdot2^n=(n-1)2^{n+1}+2$

III. $1\cdot3+3\cdot5+5\cdot7+\cdots+(2n-1)(2n+1)=\frac{n(4n^2+6n-1)}{3}$

IV. $a+(a+d)+(a+2d)+\cdots+[a+(n-1)d]=\frac{n}{2}[2a+(n-1)d]$

V. $4+8+12+\cdots+4n=2n(n+1)$

VI. $\left(1+\frac{1}{3}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\cdots\left[1+\frac{2n+1}{n^2}\right]=(n+1)^2$

VII. P.T. $2\cdot7^n+3\cdot5^n-5$ is divisible by 24

VIII. P.T. $10^n+3\cdot4^{n+2}+5$ is div. by 9.

IX. P.T. $4^n+15n-1$ is divisible by 9.

X. P.T. 3^{2n} when divided by 8, the remainder is always 1

XI. P.T. $11^{n+2}+12^{2n+1}$ is divisible by 133

XII. P.T. $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\cdots+\frac{1}{n^2}<2-\frac{1}{n}$

XIII. P.T. $1+2+3+\cdots+n<\frac{(2n+1)^2}{8}$

XIV. P.T. $(n+3)^2\le2^{n+3}$

XV. P.T. $(1+x)^n\ge1$

XVI. P.T. $n<2^n$

XVII. P.T. $1^2+2^2+3^2+\cdots+n^2>\frac{n^3}{3}$

56. (i) $\int \frac{x^2}{(1+x^3)(2+x^3)}\,dx$ (ii) $\int \frac{dx}{x(x^n+1)}$

57. (i) $\int \frac{dx}{e^x-1}$ (ii) $\int \frac{e^x}{(1+e^x)(2+e^x)}\,dx$

58. (i) $\int \frac{x^2}{(x^2+a^2)(x^2+b^2)}\,dx$ *(Exemplar)* (ii) $\int \frac{dx}{x^4-1}$

59. (i) $\int \log(2+x^2)\,dx$ (ii) $\int \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx$

60. (i) $\int (\sin^{-1}x)^2\,dx$ (ii) $\int x(\log x)^2\,dx$

61. (i) $\int \sec^3 x\,dx$ (ii) $\int e^{ax}\cos bx\,dx$

62. (i) $\int x\sec^2 x\,dx$ (ii) $\int \frac{\sqrt{x^2+1}\,(\log(x^2+1)-2\log x)}{x^4}\,dx,\ x>0$

63. (i) $\int \frac{x\cos^{-1}x}{\sqrt{1-x^2}}\,dx$ (ii) $\int \tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)dx$

64. (i) $\int \cos(\log x)\,dx$ (ii) $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ *(Exemplar)*

65. (i) $\int \frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\,dx$ *(Exemplar)* (ii) $\int \tan^{-1}\sqrt{x}\,dx$ *(Exemplar)*

66. (i) $\int \frac{2-x}{(1-x)^2}e^x\,dx$ (ii) $\int \frac{xe^{2x}}{(1+2x)^2}\,dx$

67. (i) $\int x\sqrt{x^4-1}\,dx$ (ii) $\int x\cos^{-1}x\,dx$

68. (i) $\int \sqrt{10-4x+4x^2}\,dx$ *(Exemplar)* (ii) $\int \sqrt{2ax-x^2}\,dx$ *(Exemplar)*

69. $\int \frac{\sin^{-1}\sqrt{x}-\cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x}+\cos^{-1}\sqrt{x}}\,dx,\ x\in[0,\ 1]$

70. If $\int \frac{3e^x-5e^{-x}}{4e^x+5e^{-x}}\,dx = ax+b\log|4e^x+5e^{-x}|+C$, then find a and b. *(Exemplar)*

71. If $\int \frac{x^3}{\sqrt{1+x^2}}\,dx = a(1+x^2)^{\frac{3}{2}}+b\sqrt{1+x^2}+C$, then find a and b. *(Exemplar)*

72. If $\int \frac{dx}{(x+2)(x^2+1)} = a\log|1+x^2|+b\tan^{-1}x+\frac{1}{5}\log|x+2|+C$, then find a and b. *(Exemplar)*

6 MARKS QUESTIONS

Evaluate the following integrals:

73. $\int \frac{1}{x^4+1}\,dx$ 74. $\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$ *(Exemplar)*

75. $\int e^{-3x}\cos^3 x\,dx$ *(Exemplar)* 76. $\int \sqrt{\tan x}\,dx$